QCD critical point and event-by-event fluctuations in heavy ion collisions

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A summary of work done in collaboration with K. Rajagopal and E. Shuryak. We show how heavy ion collision experiments, in particular, event-by-event fluctuation measurements, can lead to the discovery of the critical point on the phase diagram of QCD.

1. Introduction

The goal of this work is to motivate a program of heavy ion collision experiments aimed at discovering an important qualitative feature of the QCD phase diagram — the critical point at which a line of first order phase transitions separating quark-gluon plasma from hadronic matter ends [1] (see Fig. 1). The possible existence of such an endpoint E has recently been emphasized and its universal critical properties have been described [2,3]. The point E can be thought of as a descendant of a tricritical point in the phase diagram for 2-flavor QCD with *massless* quarks. As pointed out in [1], observation of the signatures of freezeout near E would confirm that heavy ion collisions are probing above the chiral transition region in the phase diagram. Furthermore, we would learn much about the qualitative landscape of the QCD phase diagram.

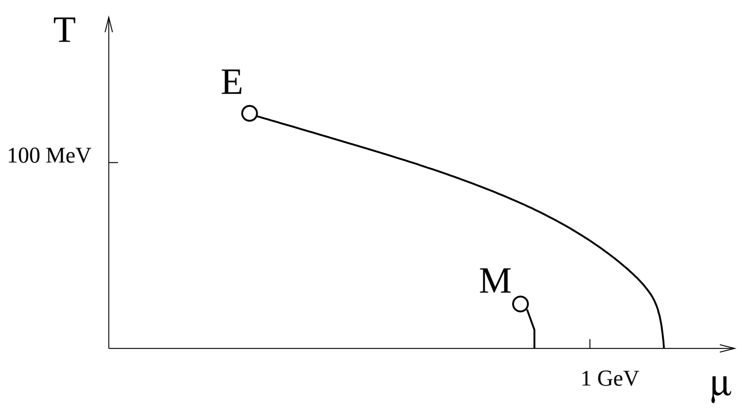

Figure 1. A “minimal” phase diagram of QCD [1]. Point E is the critical end-point of the first order phase transition separating quark-gluon and hadronic phases. Only the basic features relevant to this work are indicated, a much more complicated phase diagram is possible.

The basic ideas for observing the critical endpoint proposed in [1] are based on the fact that such a point is a genuine thermodynamic singularity at which susceptibilities diverge and the order parameter fluctuates on long wavelengths. The resulting signatures all share one common property: they are *nonmonotonic* as a function of an experimentally varied parameter such as the collision energy, centrality, rapidity or ion size. The goal of [4] is to develop a set of tools which will allow heavy ion collision experiments to discover the critical endpoint through the analysis of the variation of event-by-event fluctuations as control parameters are varied.

2. Non-critical fluctuations and comparison with data.

Before we can achieve our goal we must develop sufficient understanding of *non-critical* event-by-event fluctuations. Large acceptance detectors, such as NA49 and WA98 at CERN, have made it possible to measure important average quantities in single heavy ion collision events, such as, for example, the mean transverse momentum of the charged particles in a single event. The most remarkable property of the data is that the event-by-event distributions of such observables are as perfect Gaussians as the data statistics allow.

Our first step is to analyze the NA49 data and compare it with thermodynamic predictions for non-critical fluctuations. We find that the data is broadly consistent with the hypotheses that most of the fluctuations are thermodynamic in origin, and that PbPb collisions at 160 AGeV do not freeze out near the critical point. This allows us to establish the background, on top of which the effects of critical fluctuations should be sought as the control parameters are varied.

Most of our analysis is applied to the fluctuations of the observables characterizing the multiplicity and momenta of the charged pions in the final state of a heavy ion collision. We model the hadronic matter at freeze-out by a resonance gas in thermal equilibrium. Our simulation [4] shows that more than half of all observed pions come from resonance decays. The resonances also have a dramatic effect on the size of the multiplicity, N, fluctuations. We find:

$$\frac{\langle(\Delta N)^2\rangle}{\langle N\rangle} \approx 1.5 \ , \tag{1}$$

which is larger than the ideal gas value of 1. The contribution of resonances is important to bring this number up. The experimental value from NA49 of this ratio is 2.0. There is clearly room for non-thermodynamic fluctuations, such as fluctuations of impact parameter. Their effect can be studied and separated by varying the centrality cut using the zero degree calorimeter.

Fluctuations of *intensive* observables, such as mean p_T are less sensitive to impact parameter fluctuations. However, the effects of the flow on p_T are large and complicate the analysis. The quantity we compare with the data is the ratio: $v_{\rm inc}(p_T)/\langle p_T\rangle$, of the variance of the inclusive distribution to all-event mean p_T. The effects of the flow, which we do not calculate, should largely cancel in this ratio. We find:

$$\frac{v_{\rm inc}(p_T)}{\langle p_T\rangle} = 0.68. \tag{2}$$

The experimental value obtained from NA49 data is 0.75. We see that the major part of the observed fluctuation in p_T is accounted for by the thermodynamic fluctuations. A large potential source of the discrepancy is the "blue shift" approximation we used and could be remedied by a better treatment of flow.

A very important feature in the data is the value of the ratio of the scaled event-by-event variation to the variance of the inclusive distribution:

$$F = \frac{\langle N \rangle v^2_{\rm ebe}(p_T)}{v^2_{\rm inc}(p_T)} = 1.004 \pm 0.004. \tag{3}$$

This is a remarkable fact, since the contribution of the Bose enhancement to this ratio is almost an order of magnitude larger than the statistical uncertainty. Some mechanism must compensate for the Bose enhancement. In the next section we find a possible origin of this effect: anti-correlations due to energy conservation and thermal contact between the observed pions and the rest of the system at freeze-out.

3. Energy Conservation and Thermal Contact

We consider the effect of the energy conservation and thermal contact between the subsystem we observe, which we call B and which consists mainly of charged pions, and the remaining unobserved part of the system, which we call A and which includes the neutral pions, the resonances, the pions not in the experimental acceptance and, if the freeze-out occurs near critical point, the order parameter or sigma field. We quantify the effect by calculating the "master correlator":

$$\langle \Delta n_p \Delta n_k \rangle = v_p^2 \delta_{pk} - \frac{v_p^2 \epsilon_p v_k^2 \epsilon_k}{T^2(C_A + C_B)} , \tag{4}$$

where n_p are the pion momentum mode occupation numbers, $v_p^2 = \langle n_p \rangle (1 + \langle n_p \rangle)$, and $C_{A,B}$ are the heat capacities of the two systems A and B.

Using this expression for the correlator we can now calculate the effect of thermal contact and energy conservation on fluctuations of various observables, such as mean p_T, for example. In particular, we find that the anti-correlation introduced by this effect reduces the value of the ratio F defined in (3) by and amount comparable to the Bose enhancement effect, and thus can compensate it. This effect can be distinguished from other effects, e.g., finite two-track resolution, also countering the Bose enhancement, by the specific form of the microscopic correlator (4). The effect of energy conservation and thermal contact introduces an *off-diagonal* (in pk space, and also in isospin space) anti-correlation. Some amount of such anti-correlation is indeed observed in the NA49 data. Another important point of (4) is that as the freeze-out approaches the critical point and C_A becomes very large the anti-correlation due to energy conservation disappears.

4. Pions Near the Critical Point: Interaction with the Sigma Field

Finally, in this section, unlike the previous sections, we consider the situation in which the freeze-out occurs very close to the critical point. This point is characterized by large long-wavelength fluctuations of the sigma field (chiral condensate). We must take into account the effect of the $G\sigma\pi\pi$ interaction between the pions and such a fluctuating field.

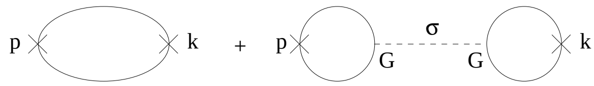

Figure 2. Diagrammatic representation of the right hand side of the correlator (5).

We do this by calculating the contribution of this effect to the “master correlator”. We find:

$$\langle \Delta n_p \Delta n_k \rangle = v_p^2 \delta_{pk} + \frac{1}{m_\sigma^2} \frac{G^2}{T} \frac{v_p^2 v_k^2}{\omega_p \omega_k}. \tag{5}$$

We see that the exchange of quanta of the soft sigma field (see Fig. 2) leads to a dramatic off-diagonal correlation, the size of which grows as we approach the critical point and m_σ decreases. This correlation takes over the off-diagonal anti-correlation discussed in the previous section.

To quantify the effect of this correlation we computed the contribution to the ratio F (3) from (5). We find:

$$\Delta F_\sigma = 0.14 \left(\frac{G_{\mathrm{freeze-out}}}{300 \text{ MeV}} \right)^2 \left(\frac{\xi_{\mathrm{freeze-out}}}{6 \text{ fm}} \right)^2 \qquad \text{for } \mu_\pi = 0 \ , \tag{6}$$

This effect, similarly to the Bose enhancement, is sensitive to over-population of the pion phase space characterized by μ_π and increases by a factor 2.5 for $\mu_\pi = 60$ MeV. We estimate the size of the coupling G to be around 300 MeV near point E, and the mass m_σ, bound by finite size effects, to be less than 6 fm. The effect (6) can thus easily exceed the present statistical uncertainty in the data (3) by 1-2 orders of magnitude.

It is also important to note that we have calculated the effect of critical fluctuations on F because this ratio is being measured in experiments, such as NA49. This observable is not optimized for detection of critical fluctuations. Observables which are more sensitive to small p_T than F (e.g., “soft F”), and/or observables which are sensitive to *off-diagonal* correlations in pk space would show even larger effect as the critical point is approached.

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